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Braneworld cosmology in (anti)-de Sitter Einstein-Gauss-Bonnet-Maxwell gravity

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Abstract: Braneworld cosmology for a domain wall embedded in the charged (anti)-de Sitter-Schwarzschild black hole of the five-dimensional Einstein-Gauss-Bonnet-Maxwell theory is considered. The effective Friedmann equation for the brane is derived by introducing the necessary surface counterterms required for a well-defined variational principle in the Gauss-Bonnet theory and for the finiteness of the bulk space. The asymptotic dynamics of the brane cosmology is determined and it is found that solutions with vanishingly small spatial volume are unphysical. The finiteness of the bulk action is related to the vanishing of the effective cosmological constant on the brane. An analogy between the Friedmann equation and a generalized Cardy-Verlinde formula is drawn.

Keywords: D-branes, AdS-CFT and dS-CFT Correspondence, Black Holes.

Contents

1. Introduction

The holographic principle in string/M-theory is formulated in terms of the AdS/CFT or dS/CFT correspondences [1, 2, 3]. These correspondences have bridged the gap between previously distinct branches of high energy physics. In particular, it has become clear that the physics of higher-dimensional black holes is closely related to that of early universe cosmology. The possibility that our observable universe may be viewed as a domain wall or 'brane' living on the boundary of a higher-dimensional black hole has recently been extensively discussed [4]–[8]. In extensions to the second Randall-Sundrum model [5], for example, the brane is embedded in a five-dimensional Schwarzschild-anti de Sitter (SAdS) bulk and the mass of the black hole induces a 'dark radiation' term into the effective Friedmann equation on the brane [9]. This term alters the asymptotic behaviour of the cosmic dynamics at high energies [10]. Thus far, attention has focused primarily on the case where the higher-dimensional bulk is represented by an AdS black hole (or more simply pure AdS space). However, the recently proposed dS/CFT correspondence provides motivation for considering the scenario where the bulk is (asymptotically) a de Sitter black hole [2, 3].

In an attempt to place the braneworld scenario in a more string theoretic setting, a number of authors have considered the effects of introducing higher-derivative terms in the curvature [11]–[17]. Within the context of the AdS/CFT correspondence, higher curvature terms in the bulk theory can arise as next-to-leading order corrections in the $1/N$ expansion of the conformal field theory (CFT) in the limit of strong coupling [18]. The Gauss Bonnet (GB) combination of curvature invariants is of particular interest to the braneworld cosmological scenario [13, 15, 16, 17]. This combination appears naturally in the next-to-leading order term of the heterotic string effective action [19, 20]. Although it contains higher-derivative gravitational terms in the metric, it is the unique combination in five dimensions that results in second-order field equations, albeit of a much more complicated form than that of standard Einstein gravity [21]. Furthermore, from the point of view of the Randall-Sundrum scenario [5], this property is crucial in ensuring a localization of gravity on the brane [14]. The singular source associated with the brane manifests itself as a δ-function in the energy-momentum tensor and this must be cancelled, at the level of the field equations, by second derivatives in the metric.

In view of the above developments, therefore, the study of black holes and braneworlds in the GB theory is well motivated, both from the field theoretic and cosmological points of view. Recently, Cai found a class of topological black holes in D-dimensional Einstein-GB theory with a cosmological constant [22], generalizing an earlier solution due to Boulware and Deser [20]. These solutions were further generalized to a class of charged SAdS and Schwarzschild-de Sitter (SdS) black holes in the Einstein-GB-Maxwell theory, where a non-trivial electromagnetic field is present [23]. In this paper, we investigate the dynamics of four-dimensional vacuum branes embedded in this class of five-dimensional (anti)-de Sitter black holes.

The structure of the paper is as follows. We present the bulk black hole solutions and the general form of the braneworld Friedmann equation in section 2. We proceed in section 3 to derive the effective Friedmann equation on the brane for the case where the higher-dimensional spacetime is asymptotically AdS. We determine the asymptotic behaviour of the brane cosmologies in section 4 for this case. In section 5, we consider the corresponding scenario where the bulk space is asymptotically de Sitter. Section 6 contains a discussion on the (generalized) Cardy-Verlinde formulae [24, 25] in both Einstein and higher-derivative theories of gravity with a cosmological constant. We conclude with a discussion in section 7.

2. Bulk black hole geometry and brane dynamics

The $(d+1)$-dimensional Einstein-GB-Maxwell bulk action has a matter sector consisting of a one-form gauge potential, A_μ, with an antisymmetric electromagnetic field strength, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, and a higher-order GB term in the gravitational sector of the theory. The action is given by[1]

$$S = \int d^{d+1}x\sqrt{-g}\left\{\frac{1}{\kappa^2}R - \Lambda + c\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\xi\sigma}R^{\mu\nu\xi\sigma}\right) - \frac{1}{4}g^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma}\right\}, \tag{2.1}$$

[1]In this paper, upper case Latin indices run from $(A, B) = (1, 2, 3)$ over the spatial sections of the world-volume of the brane, lower case Latin indices span the world-volume, $(i, j) = (0, 1, 2, 3)$, lower case Greek indices span the bulk coordinates, z is the component associated with the fifth dimension and a comma denotes partial differentiation.

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where c is an arbitrary coupling constant, κ^2 parametrizes the $(d+1)$-dimensional Planck mass, the Riemann tensor, $R_{\mu\nu\xi\sigma}$, and its contractions are constructed from the metric, $g_{\mu\nu}$, and its derivatives, $g \equiv \det g_{\mu\nu}$ and Λ represents the bulk cosmological constant.

By extremising the variations of the action (2.1) with respect to the metric tensor and gauge field, respectively, we obtain the field equations

$$\begin{aligned} 0 = &\frac{1}{2}g^{\mu\nu}\left\{c\left(R^2 - 4R_{\rho\sigma}R^{\rho\sigma} + R_{\rho\lambda\xi\sigma}R^{\rho\lambda\xi\sigma}\right) + \frac{1}{\kappa^2}R - \Lambda\right\} + \\ &+c\left(-2RR^{\mu\nu} + 4R^{\mu}{}_{\rho}R^{\nu\rho} + 4R^{\mu\rho\nu\sigma}R_{\rho\sigma} - 2R^{\mu\rho\sigma\tau}R^{\nu}{}_{\rho\sigma\tau}\right) - \\ &-\frac{1}{\kappa^2}R^{\mu\nu} + \frac{1}{2}\left(F^{\mu\sigma}F^{\nu}{}_{\sigma} - \frac{1}{4}F_{\sigma\rho}F^{\sigma\rho}g^{\mu\nu}\right) \end{aligned} \quad (2.2)$$

$$\partial_\nu\left(\sqrt{-g}F^{\mu\nu}\right) = 0\,. \quad (2.3)$$

We consider the case where the bulk spacetime corresponds to a static, hyper-spherically symmetric geometry with a line element given by

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2\sum_{A,B=1}^{d-1}\tilde{g}_{AB}dx^A dx^B\,, \quad (2.4)$$

where $\{\nu(r), \lambda(r)\}$ are functions of the radial coordinate, r, and the metric $\tilde{g}_{ij}$ is the metric of the $(d-1)$-dimensional Einstein manifold with a Ricci tensor defined by $\tilde{R}_{ij} = kg_{ij}$. The constant k has values $k = \{d-2, 0, -(d-2)\}$ for a $(d-1)$-dimensional unit sphere, a flat euclidean space and a $(d-1)$-dimensional unit hyperboloid, respectively.

Restricting to the five-dimensional case ($d = 4$), eqs. (2.2) and (2.3) admit the charged black hole solution [23]:

$$\begin{aligned} e^{2\nu} &= e^{-2\lambda} \\ &= \frac{1}{2c}\left\{ck + \frac{r^2}{2\kappa^2} \pm \sqrt{\frac{r^4}{4\kappa^4}\left(\frac{4c\kappa^2}{l^2} - 1\right)^2 - \frac{2c\mu}{\kappa^2}\left(\frac{4c\kappa^2}{l^2} - 1\right) - \frac{2cQ^2}{3r^2}}\right\}, \end{aligned} \quad (2.5)$$

$$F_{tr} = \frac{Q}{r^3}\,, \quad (2.6)$$

where the constants $\{Q, \mu\}$ are related to the charge and gravitational mass of the black hole, respectively, and

$$\frac{1}{l^2} \equiv \frac{1}{4c\kappa^2}\left(1 \pm \sqrt{1 + \frac{2c\Lambda\kappa^4}{3}}\right). \quad (2.7)$$

The constant, l^2, is determined by the Gauss-Bonnet coupling parameter, c, and the bulk cosmological constant, Λ. It corresponds to the length parameter of the asymptotically AdS space when r is large. If $c\Lambda > 0$, l^2 can be formally negative and the spacetime then becomes asymptotically de Sitter. (In this case, it represents a charged de Sitter space or the charged Nariai (SdS) black hole. See ref [23] for details). In principle, the above solution (2.5)–(2.6) for positive l^2 generalizes the well-known Reissner-Nordstrom-AdS black hole solution to Einstein-GB-Maxwell gravity. Moreover, it may also correspond to pure, charged AdS space and it has been observed that Hawking-Page phase transitions [26] between the SAdS black hole and pure AdS space can be realized [23]. We refer to the two solutions in eq. (2.5) as the 'positive-' and 'negative-branch' solutions, respectively. They reduce to the class considered recently by Cai in the charge neutral limit ($Q = 0$) [22].

We now proceed to consider the motion of a domain wall (three-brane) along a timelike geodesic of the five-dimensional, static background defined by eqs. (2.4) and (2.5). The equation of motion of the brane is interpreted by an observer confined to the brane as an effective Friedmann equation describing the expansion or contraction of the universe. From this Friedmann equation, we can deduce the energy and entropy of the matter in the brane universe. Specifically, we consider a brane action of the form:

$$S_{\rm br} = -\eta \int d^4x \sqrt{-h}\,, \tag{2.8}$$

where η is a positive constant representing the tension associated with the brane and h is the determinant of the boundary metric, h_{ij}, induced by the bulk metric, $g_{\mu\nu}$.

We employ the method developed in ref [12] to derive the Friedmann equation. The metric (2.4) is rewritten by introducing new coordinates (y, τ) and a scalar function $A = A(y, \tau)$ that satisfies the set of constraint equations:

$$\begin{aligned} l^2 e^{2A+2\lambda} A_{,y}^2 - e^{-2\lambda} t_{,y}^2 &= 1\,, \\ l^2 e^{2A+2\lambda} A_{,y} A_{,\tau} - e^{-2\lambda} t_{,y} t_{,\tau} &= 0\,, \\ l^2 e^{2A+2\lambda} A_{,\tau}^2 - e^{-2\lambda} t_{,\tau}^2 &= -l^2 e^{2A}\,, \end{aligned} \tag{2.9}$$

where a comma denotes partial differentiation. When $\lambda = -\nu$, as in eq. (2.5), the metric (2.4) may then be written in the form

$$ds^2 = dy^2 + e^{2A(y,\tau)} \sum_{i,j=1}^{4} \tilde{g}_{ij} dx^i dx^j\,, \tag{2.10}$$

where $r = l \exp(A)$. The non-trivial Riemann components and its contractions for this metric are shown in the appendix. Since we are interested in the cosmological implications, we assume that the metric, $\tilde{g}_{ij}$, respects the same symmetries as the metric of the Friedmann-Robertson-Walker (FRW) models, i.e., we assume that

$$\tilde{g}_{ij} dx^i dx^j \equiv l^2 \left(-d\tau^2 + d\Omega_{k,3}^2\right)\,, \tag{2.11}$$

where $d\Omega_{k,3}^2$ is the metric of unit three-sphere for $k > 0$, three-dimensional euclidean space for $k = 0$ and the unit three-hyperboloid for $k < 0$. Thus, choosing a timelike coordinate, $\tilde{t}$, such that $d\tilde{t} \equiv l e^A d\tau$, implies that the induced metric on the brane takes the FRW form:

$$ds_{\rm brane}^2 = -d\tilde{t}^2 + l^2 e^{2A} d\Omega_{k,3}^2\,. \tag{2.12}$$

It follows by solving eqs. (2.9) that

$$H^2 = A_{,z}^2 - \frac{\mathrm{e}^{-2\lambda}\mathrm{e}^{-2A}}{l^2}\,, \tag{2.13}$$

where the Hubble parameter on the brane is defined by $H \equiv dA/d\tilde{t}$. Thus, for a vacuum brane that has no matter confined to it, the cosmic expansion (contraction) is determined once the functional forms of $\{A, \lambda\}$ have been determined.

3. Brane dynamics in asymptotically AdS space

In this section we consider the brane equation of motion in an asymptotically AdS space, corresponding to $l^2 > 0$. We defer the case $l^2 < 0$ to section 5.

The functional form of $A_{,z}$ is deduced by deriving the surface counterterms in the Gauss-Bonnet theory (2.1) that generalize the Gibbons-Hawking term in Einstein gravity [27]. This has been done previously in ref [23] and we briefly summarize the approach here. The reader is referred to [23] for details. Auxiliary fields, F, $B_{\mu\nu}$ and $C_{\mu\nu\rho\sigma}$ are introduced and the Gauss-Bonnet sector of the action (2.1) is rewritten in terms of these variables by employing the field equations. Imposing a Dirichlet type of boundary condition allows appropriate boundary conditions to be imposed on the scalar quantities. This in turn implies that the action can be expressed in terms of a bulk part and a surface term. The variational principle then becomes well defined if an appropriate boundary term is introduced. An important point is that the complete boundary action does not acquire terms from the form field, since the bulk action for the form field does not contain second- or higher-order derivatives. Thus, we may extract the relevant equations from [23], specialized to the Gauss-Bonnet interaction. In particular, the counterterm is identical to eq. (130) of [23]:

$$\begin{aligned} S_b = \int d^4x\sqrt{-h}\Big[& 4c\nabla_\mu n^\mu F - 8c\left(n_\mu n_\nu \nabla_\sigma n^\sigma + \nabla_\mu n_\nu\right) B^{\mu\nu} + \\ & + 8c n_\mu n_\nu \nabla_\tau n_\sigma C^{\mu\tau\nu\sigma} + \frac{2}{\kappa^2}\nabla_\mu n^\mu - \eta\Big]\,, \end{aligned} \tag{3.1}$$

where n^μ is the unit vector perpendicular to the boundary and we have also included the Gibbons-Hawking term and the counterterm (2.8) arising from the vacuum energy on the brane. Extremizing the variation over F of the full action[2] $(S + 2S_b)|_{z=z_0}$ then implies that

$$\begin{aligned} 0 = {} & 16cR_{,z} - 8c\left(4R_{zz,z} + R_{i\ ,z}^{\ i}\right) + 8cC_{ziz\ ,z}^{\ \ \ i} + \\ & + \left(48cF - 192cB_{zz} + c\left(56R_{zz} - 8R_i^{\ i}\right) + \frac{24}{\kappa^2}\right)A_{,z} + 4\eta\,. \end{aligned} \tag{3.2}$$

It is important to emphasize that the boundary terms are derived from the internal consistency of the theory and are not simply introduced in an *ad hoc* fashion.

[2] A factor of two arises because two bulk spacetimes with a common boundary are being considered. In [5], two copies of the AdS spaces are glued by a brane by imposing the Z_2-symmetry. In deriving the corresponding Friedmann equation in [12], essentially only one side was considered. In the situation that we are considering above, there are two copies of the AdS space as in [5], and we therefore need twice the surface counterterms.

When the metric has the form (2.4) with $\lambda = -\nu$, the Ricci scalar and non-trivial components of the Ricci tensor are given by

$$
\begin{aligned}
R =& -\left(\mathrm{e}^{2\nu}\right)'' - \frac{6}{r}\left(\mathrm{e}^{2\nu}\right)' - \frac{6}{r^2}\mathrm{e}^{2\nu} + \frac{3k}{r^2} \\
=& \frac{5}{\epsilon l^2} \mp \frac{1}{2cr^6}\Bigg\{2\tilde{Q}^4 + 18\epsilon\tilde{\mu}(4\epsilon-1)\tilde{Q}^2r^2 + 6\left(2\epsilon\tilde{\mu}\left(4\epsilon-1\right)\right)^2r^4 - \\
&\qquad - \frac{31\left(4\epsilon-1\right)^2}{4\kappa^4}\tilde{Q}^2r^6 - \frac{15\left(4\epsilon-1\right)^3\epsilon\tilde{\mu}}{\kappa^4}r^8 + 20\left(\frac{\left(4\epsilon-1\right)^2}{4\kappa^4}\right)^2 r^{12}\Bigg\} \times \\
&\times\left\{-\left(2\epsilon\tilde{\mu}\left(4\epsilon-1\right)\right) - \frac{\tilde{Q}^2}{r^2} + \frac{\left(4\epsilon-1\right)^2}{4\kappa^4}r^4\right\}^{-3/2}
\end{aligned}
\tag{3.3}
$$

and

$$
\begin{aligned}
R_{zz} =& \,\mathrm{e}^{2\nu}R_{rr} = -\frac{1}{2}\left(\mathrm{e}^{2\nu}\right)'' - \frac{3}{2r}\left(\mathrm{e}^{2\nu}\right)' \\
=& -\frac{1}{\epsilon l^2} \mp \frac{1}{2cr^6}\left\{-\tilde{Q}^4 - \frac{4\left(4\epsilon-1\right)^2}{\kappa^4}\tilde{Q}^2r^6 - \frac{6\left(4\epsilon-1\right)^3\epsilon\tilde{\mu}}{\kappa^4}r^8 + 8\left(\frac{\left(4\epsilon-1\right)^2}{4\kappa^4}\right)^2 r^{12}\right\} \times \\
&\times\left\{-2\epsilon\tilde{\mu}\left(4\epsilon-1\right) - \frac{\tilde{Q}^2}{r^2} + \frac{\left(4\epsilon-1\right)^2}{4\kappa^4}r^4\right\}^{-3/2} .
\end{aligned}
\tag{3.4}
$$

respectively, where a prime denotes differentiation with respect to r, we have defined the rescaled parameters

$$
\epsilon \equiv \frac{c\kappa^2}{l^2}\,, \qquad \tilde{\mu} \equiv \frac{l^2\mu}{\kappa^4}\,, \qquad \tilde{Q}^2 \equiv \frac{2c}{3}Q^2 \tag{3.5}
$$

to simplify the notation and the coordinates z and r are related to each other by $dz = \mathrm{e}^{-\nu}dr$. Substituting eqs. (3.3) and (3.4) into eq. (3.2) then yields a relationship between $A_{,z}$ and the parameters of the bulk solution:

$$
\begin{aligned}
0 =& \; 4\eta \pm \frac{12}{r^7}\left\{3\tilde{Q}^4 + 16\epsilon^2\tilde{\mu}^2(4\epsilon-1)^2r^4 + 6\tilde{Q}^2r^2\left(2\epsilon\tilde{\mu}(4\epsilon-1) + \frac{(4\epsilon-1)^2}{4\kappa^4}r^4\right)\right\} \times \\
&\times\left\{-2c\tilde{\mu}(4\epsilon-1) - \frac{\tilde{Q}^2}{r^2} + \frac{(4\epsilon-1)^2}{4\kappa^4}r^4\right\}^{-3/2} \times \\
&\times\left[\frac{k}{2} + \left(\frac{1}{4\epsilon l^2}\right)r^2 \pm \frac{1}{2c}\left\{-2\epsilon\tilde{\mu}(4\epsilon-1) - \frac{\tilde{Q}^2}{r^2} + \frac{(4\epsilon-1)^2}{4\kappa^4}r^4\right\}^{1/2}\right]^{1/2} + \\
&+\Bigg[-24c\Bigg\{\frac{3}{\epsilon l^2} \pm \frac{1}{2cr^6}\Bigg\{6\tilde{Q}^4 + 3\tilde{Q}^2\left(10\epsilon\tilde{\mu}(4\epsilon-1)r^2 - \frac{3(4\epsilon-1)^2}{4\kappa^4}r^6\right) + \\
&\qquad + 2r^4\left(5\left(2\epsilon\tilde{\mu}\left(4\epsilon-1\right)\right)^2 + 6\left(\frac{(4\epsilon-1)^2}{4\kappa^4}\right)^2r^8 - \frac{9(4\epsilon-1)^3\epsilon\tilde{\mu}}{2\kappa^4}r^4\right)\Bigg\} \\
&\qquad \times\left\{-2\epsilon\tilde{\mu}(4\epsilon-1) - \frac{\tilde{Q}^2}{r^2} + \frac{(4\epsilon-1)^2}{4\kappa^4}r^4\right\}^{-3/2}\Bigg\} + \frac{24}{\kappa^2}\Bigg]A_{,z}\,.
\end{aligned}
\tag{3.6}
$$

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It now only remains to identify the scale factor of the metric on the brane, $a(\tilde{t}) \equiv l e^A$, with the radial coordinate, r, of the brane in the bulk metric (2.4):

$$a = r(\tilde{t}) = l e^{A(\tilde{t})} . \tag{3.7}$$

By combining eqs. (2.13), (3.6) and (3.7), we arrive at the Friedmann equation describing the motion of the 3-brane in the bulk spacetime. We find it convenient to express this equation in the compact form

$$H^2 = \frac{\mathcal{G}^2}{\mathcal{H}^2} - \frac{X(a)}{a^2} , \tag{3.8}$$

where

$$\mathcal{G} = 4\eta \pm \frac{12 X^{1/2}}{Y^{3/2} a^7} \left\{ 3\tilde{Q}^4 + 16\epsilon^2 \tilde{\mu}^2 (4\epsilon - 1)^2 a^4 + 6\tilde{Q}^2 a^2 \left(2\epsilon\tilde{\mu}(4\epsilon - 1) + \frac{(4\epsilon - 1)^2}{4\kappa^4} a^4 \right) \right\} \tag{3.9}$$

and

$$\begin{aligned} \mathcal{H} = &-\frac{48}{\kappa^2} \mp \frac{12}{Y^{3/2} a^6} \left\{ 6\tilde{Q}^4 + 3\tilde{Q}^2 \left(10\epsilon\tilde{\mu}(4\epsilon - 1)a^2 - \frac{3(4\epsilon - 1)^2}{4\kappa^4} a^6 \right) + \right. \\ &\left. + 2a^4 \left(5\left(2\epsilon\tilde{\mu}(4\epsilon - 1)\right)^2 + 6\left(\frac{(4\epsilon - 1)^2}{4\kappa^4} \right)^2 a^8 - \frac{9(4\epsilon - 1)^3 \epsilon\tilde{\mu}}{2\kappa^4} a^4 \right) \right\} \end{aligned} \tag{3.10}$$

and we have identified the two important functions of the scale factor:

$$X \equiv \frac{k}{2} + \frac{a^2}{4\epsilon l^2} \pm \frac{Y^{1/2}}{2c} , \tag{3.11}$$

$$Y \equiv -2\epsilon\tilde{\mu}(4\epsilon - 1) - \frac{\tilde{Q}^2}{a^2} + \frac{(4\epsilon - 1)^2}{4\kappa^4} a^4 . \tag{3.12}$$

In the following section, we proceed to analyze the qualitative early- and late-time behaviour of the cosmic dynamics on the brane.

4. Qualitative dynamics of the brane cosmology

The simplest case to consider arises when the black hole has vanishing mass and charge. In this case, the Friedmann equation (3.8) simplifies to

$$H^2 = \frac{\eta^2 \kappa^4}{36} \left[2 \pm 3|4\epsilon - 1| \right]^{-2} - \frac{k}{2a^2} - \frac{1}{4\epsilon l^2} \left(1 \pm |4\epsilon - 1| \right) . \tag{4.1}$$

We should note that the $\pm$ signs in (4.1) correspond to the two solutions in (2.7). Formally, this is equivalent to the standard Friedmann equation, containing a cosmological constant term and a curvature term. The three parameters, $\{\eta, c, l^2\}$, in eq. (4.1) are related by requiring that the leading-order divergence in the bulk AdS action is cancelled when the brane moves to infinity in the asymptotic AdS space. The divergence is cancelled if the brane tension satisfies [12]

$$\frac{\eta}{l} = \frac{72c}{l^4} - \frac{6}{\kappa^2 l^2} . \tag{4.2}$$

When no higher-order curvature invariants are included in the action (2.1), eq. (4.2) reduces to the fine-tuning condition imposed by Randall and Sundrum for the vacuum brane to respect four-dimensional Poincare invariance [5]. Once the brane tension has been specified by eq. (4.2), there are no *a priori* restrictions on the remaining two parameters, and in principle, an appropriate choice can then be made to yield an effective positive cosmological constant on the brane. Consequently, the brane can undergo a de Sitter phase of expansion (inflation).

An interesting special case arises for $l^2 = 4c\kappa^2$ ($\epsilon = 1/4$), where the two branches of the bulk solution (2.5) coincide. Eq. (4.2) then implies that $\eta l \kappa^2 = 12$ and when this expression is substituted into eq. (4.1), the effective cosmological constant *vanishes*. In this model, therefore, the vanishing of the cosmological constant is associated with the cancellation of the leading-order divergence of the action. In the limit of small Gauss-Bonnet coupling, $\epsilon \ll 1$, the brane tension also cancels the contributions from the five-dimensional cosmological constant when the negative-branch solution is considered and eq. (4.2) is satisfied. Unless otherwise stated, we consider this branch in what follows.[3]

It is of interest to determine how the expansion of the braneworld is affected by the charge and mass associated with the bulk black hole. The advantage of the parametrization (3.9)–(3.12) is that it separates the Friedmann equation (3.8) into two sectors that both admit a physical interpretation. The first term is determined by the ratio, $\mathcal{G}^2/\mathcal{H}^2$, and the numerator, eq. (3.9), contains the brane tension, η, and some further terms that depend on the parameters of the bulk black hole. In the limit of small Guass-Bonnet coupling, $\epsilon \ll 1$, and vanishing charge, we find that $\mathcal{G} = 4\eta + \mathcal{O}(\epsilon^2)$. In effect, therefore, this term parametrizes the vacuum energy of the brane. By analogy with the standard, four-dimensional Friedmann equation, the denominator, eq. (3.10), then plays the role of the effective Planck mass. This parameter is time-dependent, in general, due to the non-trivial black hole mass and charge. To lowest-order in ϵ, it also reduces to a constant, $\mathcal{H} = 24(1-12\epsilon)/\kappa^2 + \mathcal{O}(\epsilon^2)$, when the charge vanishes.

The second sector of the Friedmann equation, X/a^2, may be interpreted as a generalized curvature term. This sector depends directly on the spatial curvature of the world-volume, k, but also acquires corrections due to deviations of the bulk geometry from pure AdS space. Indeed, eq. (3.11) reduces to a constant value in the limit where the first term on the right-hand side of this equation is dominant.

When eq. (4.2) is satisfied and the black hole charge vanishes, the limit of the negative-branch Friedmann equation (3.8) as $\epsilon \to 0$ is given by

$$H^2 = -\frac{k}{2a^2} + \frac{\mu}{a^4} - \frac{2c\kappa^2\mu^2}{a^8} + \mathcal{O}(c^2)\,, \tag{4.3}$$

where we have returned to the original parameters (3.5). As in the massless case considered above, eq. (4.2) ensures that the cosmological terms are cancelled. At this order, the leading terms in eq. (4.3) correspond to those of the second Randall-Sundrum model [5], as expected. The term due to the Gauss-Bonnet sector has a strong dependence on the scale factor and is rapidly redshifted by the cosmic expansion. Introducing a bulk charge

[3] The bulk solution for the positive-branch has been shown to be unstable [20].

in this limit results in a complicated expression for the Friedmann equation, where all but one of the new terms that arise do so with a linear dependence on ϵ. The remaining term arises from the expansion of eq. (3.11) and plays the role of a shear or anisotropic stress. It corresponds to the contribution that arises when the brane is embedded in the Reissner-Nordstrom-AdS black hole bulk space [7, 28].

More generally, the functions (3.11) and (3.12) play a crucial role in determining the asymptotic behaviour of the braneworld and, in particular, they must both remain semi-positive definite if physical solutions to eq. (3.8) are to exist. (These functions are not constrained in this way in the special case where $\tilde{Q}^2 = \tilde{\mu} = 0$, since eq. (3.9) then depends only on the brane tension). We may deduce immediately, therefore, that the effective curvature term in the Friedmann equation (3.8) always acts to *reduce* the expansion rate of the brane if the black hole has a mass and/or a charge. An alternative interpretation is that the brane expands as if its effective curvature is positive and this is independent of its specific curvature, k. An important consequence of this property is that the brane may recollapse before the effective cosmological constant on the brane is able to dominate the dynamics.[4] This is true for all spatially open, flat and closed branes and therefore differs from the second Randall-Sundrum scenario based on the SAdS bulk space, where recollapse is only possible in the positively curved model [5].

In determining the asymptotic behaviour, we first consider the large world-volume limit, $a \to \infty$. It is straightforward to verify that both the mass and charge become negligible in eqs. (3.11) and (3.12) as the scale factor diverges. However, to ensure that the function (3.11) remains positive in this limit, we require that $1 \pm |4\epsilon - 1| > 0$ when $\epsilon > 0$ and $1 \pm |4\epsilon - 1| < 0$ when $\epsilon < 0$. There is no corresponding limit on the value of the black hole mass. Moreover, the leading-order term in eq. (3.9) redshifts as the eighth (sixth) power of the scale factor when the charge is trivial (non-trivial). Thus, this function tends to a constant, $\mathcal{G} \to 4\eta$, as in the massless, neutral solution. Similarly, the dominant term in eq. (3.10) is the term corresponding to $\tilde{\mu} = \tilde{Q} = 0$. The asymptotic behaviour in the region of parameter space where the scale factor can become arbitrarily large is therefore given by eq. (4.1). This behaviour can be understood from a physical point of view, since an increase in the spatial volume of the brane corresponds to the brane being located at greater distances from the event horizon of the black hole.

An ambiguity arises, however, when considering the opposite limit, where the scale factor becomes small. Let us first consider the case where the black hole is massive but has vanishing charge. The range of the Gauss-Bonnet parameter, ϵ, is important in this case. In the region of parameter space $0 < \epsilon < 1/4$, eq. (3.12) is positive-definite for all allowed values of the scale factor. Further motivation for considering this range of values for ϵ arises from the asymptotic form of the bulk solution (2.5) as r diverges. In this limit, the metric reduces to the standard Reissner-Nordstrom-AdS black hole with a real charge [23]. On the other hand, eq. (3.11) implies that for the non-positively curved brane, $k \leq 0$, it is necessary to take the positive-branch if a is to become arbitrarily small. In the negatively

[4] We are assuming implicitly in this discussion that the effective cosmological constant on the brane is positive.

curved model, the further constraint, $|\epsilon(4\epsilon-1)| > 2c^2/\tilde{\mu}$, must also be satisfied. There is no constraint for the positively curved brane on the positive branch solution, but the corresponding limit $|Y|^{1/2} < 2c$ must be satisfied for the negative-branch.

Nevertheless, even when the above necessary conditions are satisfied, we find that the ratio

$$\frac{\mathcal{G}^2}{\mathcal{H}^2} \to \frac{4X}{25a^2} \tag{4.4}$$

in the limit $a \to 0$. Substituting this limit into the Friedmann equation (3.8) then implies that $H^2 < 0$. Although this is indicative of unphysical behaviour, it is interesting because it implies that when $0 < \epsilon < 1/4$, the scale factor of the vacuum brane is bounded from below and consequently can not approach zero in a continuous fashion. It is interesting that this conclusion holds for arbitrary values of the black hole mass.

In the remaining region of parameter space, $\epsilon(4\epsilon-1) > 0$, it follows that $Y \to 0$ at some finite value of the scale factor as the spatial volume decreases, assuming implicitly that the black hole mass parameter $\tilde{\mu} > 0$. Although the Hubble parameter remains finite as $Y \to 0$, the Ricci curvature of the world-volume diverges and this corresponds to a singular initial state of finite volume. It is difficult to motivate such behaviour from a physical point of view.

To summarize, thus far, there is a lower limit to the spatial volume associated with physical cosmological solutions on the brane world-volume. This conclusion holds for all values of ϵ, and hence the Gauss-Bonnet coupling, c. It is also independent of the value of the bulk cosmological constant (modulo the restriction imposed in this section that $l^2 > 0$). This implies that the vacuum brane can not expand from a big bang initial (singular) state of vanishing spatial volume.

When the black hole charge is non-trivial, the functional forms of eqs. (3.11) and (3.12) differ in the limit of small scale factor. In particular, they are dominated by the term proportional to $\tilde{Q}^2/a^2$, indicating that a necessary condition for the brane to expand from an arbitrarily small volume is that $\tilde{Q}^2 < 0$. Since the charge of the black hole should be real, this implies that $\epsilon < 0$ ($c < 0$). In this case, it is possible to find appropriate values of the charge to ensure that Y is positive-definite. On the other hand, since eq. (3.11) becomes dominated by the third term on the right hand side, we must choose the positive branch in this limit. However, in this region of parameter space, we deduce that

$$\frac{\mathcal{G}^2}{\mathcal{H}^2} \to \frac{X}{4a^2} \tag{4.5}$$

and it follows, after substitution into eq. (3.8), that the scalar factor of the world-volume is once more bounded from below, as in the charge neutral case.

We conclude, therefore, that the charge and mass of the bulk black hole have significant effects on the asymptotic behaviour of the brane cosmology at small spatial volumes, but become negligible in the region of parameter space where expansion is unbounded from above.

Before concluding this section, we consider a second special case where the restriction on the positivity of eq. (3.11) need not be imposed. This arises when the tension of the brane

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formally vanishes, $\eta \to 0$. Although the interpretation of a vacuum brane with vanishing tension is unclear (note that the effective tension does not vanish), the function X in this case appears linearly in the Friedmann equation (3.8). Moreover, it follows from eq. (4.2) that $\epsilon = 1/12$ when the brane tension vanishes and it is interesting that this is precisely the critical value for the thermodynamical entropy associated with the bulk black hole spacetime to also vanish [23]. This value of ϵ ensures that eq. (3.12) is always positive when the black hole is neutral and we may therefore consider the small volume behaviour of the brane.[5] The limit of the Friedmann equation (3.8) is then deduced from eq. (4.4):

$$\begin{aligned} H^2 &= \frac{X}{a^2}\left(-\frac{21}{25} \mp \frac{4a^2}{25\kappa^2\sqrt{\tilde{\mu}}}\right) \\ &= -\frac{21}{25a^2}\left(\frac{k}{2} \mp \frac{\sqrt{\tilde{\mu}}}{6c}\right) - \frac{63}{25l^2} \mp \left(\frac{k}{2} \mp \frac{\sqrt{\tilde{\mu}}}{6c}\right)\frac{4}{25\kappa^2\sqrt{\tilde{\mu}}} + \mathcal{O}\left(a^2\right). \end{aligned} \tag{4.6}$$

Eq. (4.6) implies that the brane behaves as if it were dominated by its spatial curvature, in the sense that $H \propto a^{-1}$. (There is a subdominant negative cosmological constant). The specific curvature of the spatial hypersurfaces of the induced metric of the brane is uniquely specified by the sign of k. However, the brane acts as if it had *arbitrary* curvature, because the sign of the first term on the right hand side of eq. (4.6) is not specified *a priori*. It is determined by the mass of the black hole and the Gauss-Bonnet coupling.

This concludes the discussion on brane dynamics in asymptotically AdS bulk spaces. In the next section, we determine the form of the Friedmann equation when the bulk space is asymptotically de Sitter.

5. Brane dynamics in asymptotically dS space

In the case where l^2, as defined in eq. (2.7), is formally negative, the higher-dimensional bulk spacetime is asymptotically de Sitter space. In order to analyse this region of parameter space, it proves convenient to define

$$\bar{l}^2 \equiv -l^2 > 0\,. \tag{5.1}$$

If we then consider a metric of the form (2.10), where

$$\bar{g}_{ij}dx^i dx^j \equiv \bar{l}^2\left(-d\tau^2 + d\Omega_{k,3}^2\right), \tag{5.2}$$

is introduced instead of eq. (2.11), we find that the effective Hubble parameter on the brane is given by

$$H^2 = A_{,z}^2 - \frac{\mathrm{e}^{-2\lambda}\mathrm{e}^{-2A}}{\bar{l}^2}\,. \tag{5.3}$$

We may now follow the same argument as that presented in section 3 to deduce the form of $A_{,z}$. Omitting the details, we find after defining the parameters:

$$\bar{\epsilon} \equiv \frac{c\kappa^2}{\bar{l}^2}\,, \qquad \bar{\mu} \equiv \frac{\mu\bar{l}^2}{\kappa^4}\,, \qquad \bar{Q}^2 = \frac{2c}{3}Q^2 \tag{5.4}$$

[5]We do not consider the charged case, since this requires $\epsilon < 0$ for consistency.

that the Friedmann equation has the form

$$H^2 = \frac{\bar{\mathcal{G}}^2}{\bar{\mathcal{H}}^2} - \frac{\bar{X}}{a^2}\,, \tag{5.5}$$

where

$$\begin{aligned}\bar{\mathcal{G}} &= 4\eta \pm \frac{12\bar{X}^{1/2}}{\bar{Y}^{3/2}a^7}\Bigg\{3\bar{Q}^4 + 16\bar{\epsilon}^2\bar{\mu}^2\left(4\bar{\epsilon}+1\right)^2 a^4 + \\ &\qquad + 6\bar{Q}^2a^2\left(2\bar{\epsilon}\bar{\mu}\left(4\bar{\epsilon}+1\right) + \frac{\left(4\bar{\epsilon}+1\right)^2}{4\kappa^4}a^4\right)\Bigg\} \end{aligned} \tag{5.6}$$

$$\begin{aligned}\bar{\mathcal{H}} &\equiv -\frac{48}{\kappa^2} \mp \frac{12}{\bar{Y}^{3/2}a^6}\Bigg\{6\bar{Q}^4 + 3\bar{Q}^2\left(-10\bar{\epsilon}\bar{\mu}(4\bar{\epsilon}+1)a^2 - \frac{3(4\bar{\epsilon}+1)^2}{4\kappa^4}a^6\right) + \\ &\qquad + 2a^4\left(5\left(-2\bar{\epsilon}\bar{\mu}\left(4\bar{\epsilon}+1\right)\right)^2 + 6\left(\frac{\left(4\bar{\epsilon}+1\right)^2}{4\kappa^4}\right)^2 a^8 + \frac{9(4\bar{\epsilon}+1)^3\bar{\epsilon}\bar{\mu}}{2\kappa^2}a^4\right)\Bigg\}\end{aligned} \tag{5.7}$$

and

$$\bar{X} \equiv \frac{k}{2} + \frac{a^2}{4\bar{\epsilon}\bar{l}^2} \pm \frac{\bar{Y}^{1/2}}{2c} \tag{5.8}$$

$$\bar{Y} \equiv 2\bar{\epsilon}\bar{\mu}(4\bar{\epsilon}+1) - \frac{\bar{Q}^2}{a^2} + \frac{(4\bar{\epsilon}+1)^2}{4\kappa^4}a^4\,. \tag{5.9}$$

The dependence on the scale factor, a, of the Friedmann equation (5.5) is identical to that given in eq. (3.8). Thus, in general, eqs. (5.8) and (5.9) must remain positive definite and the asymptotic limit of the brane in the large volume limit is qualitatively similar to the corresponding model where the bulk space is asymptotically AdS ($l^2 > 0$). The region of parameter space where such a limit is consistent is given by $1 \pm |4\bar{\epsilon}+1| > 0$ if $\bar{\epsilon} > 0$ and $1 \pm |4\bar{\epsilon}+1| < 0$ if $\bar{\epsilon} < 0$. Finally, similar conclusions to those of previous section are drawn regarding the restrictions on the small volume limit.

The physical interpretation of the functions (5.6), (5.7) and (5.8) may also be made along similar lines to that discussed in section 4. In the next section, we consider the physical interpretation of the Friedmann equation (3.8) in more detail and investigate whether an analogy can be made between this equation and a generalized Cardy-Verlinde formula [24, 25].

6. Cosmological entropy and generalized Cardy-Verlinde formula

As we have seen in the previous sections, the GB term in action (2.1) results in a complicated form for the Friedmann equation (3.8). Recently, Verlinde [24] drew an interesting analogy between the FRW equations of a standard, closed, radiation-dominated universe and the two-dimensional entropy formula due to Cardy [25]. In this section, we will extend this analogy to the FRW equations under consideration.

Let us first investigate this analogy in Einstein gravity for the usual $(n+1)$-dimensional FRW Universe with a metric

$$ds^2 = -d\tau^2 + a^2(\tau)g_{ij}dx^i dx^j\,, \tag{6.1}$$

where the n-dimensional spatial hypersurfaces with negative, zero or positive curvature are parametrized by $k = -1, 0, 1$, respectively.[6] We limit our discussion mainly to that of the closed universe ($k = 1$), with a spatial volume defined by $V = a^n \int d^n x \sqrt{g}$. The standard FRW equations may then be written as

$$\begin{aligned} H^2 &= \frac{16\pi G}{n(n-1)}\rho - \frac{k}{a^2}, \\ \dot{H} &= -\frac{8\pi G}{(n-1)}(\rho + p) + \frac{k}{a^2}, \end{aligned} \tag{6.2}$$

where $\rho = \rho_m + \frac{\Lambda}{8\pi G}$, $p = p_m - \frac{\Lambda}{8\pi G}$, Λ is a cosmological constant and ρ_m and p_m are the energy density and pressure of the matter contributions. The energy conservation equation is

$$\dot{\rho} + n(\rho + p)\frac{\dot{a}}{a} = 0 \tag{6.3}$$

and, for a perfect fluid matter source with equation of state $p_m = \omega \rho_m$ (ω = constant), eq. (6.3) is solved as:

$$\rho = \rho_0 a^{-n(1+\omega)} + \frac{\Lambda}{8\pi G}. \tag{6.4}$$

The definitions for the Hubble, Bekenstein and Bekenstein-Hawking entropies are [24]:

$$S_H = (n-1)\frac{HV}{4G}, \qquad S_{BH} = (n-1)\frac{V}{4Ga}, \qquad S_B = \frac{2\pi a}{n}E, \tag{6.5}$$

where the total energy, E, is defined as $E = \rho V$ and contains the contribution from the cosmological constant term. This differs from that of the standard case, where the definitions of the entropies S_{BH} and S_B may differ slightly in their coefficients. This is specific to the presence of a cosmological constant [30, 31].

By employing the above definitions (6.5), one can easily rewrite the FRW equations (6.2) as a cosmological Cardy-Verlinde (CV) formula:

$$\begin{aligned} S_H &= \frac{2\pi}{n} a \sqrt{E_{BH}\left(2E - kE_{BH}\right)}, \\ kE_{BH} &= n\left(E + pV - T_H S_H\right), \end{aligned} \tag{6.6}$$

where the energy and Hawking temperature of the black hole are defined as

$$E_{BH} = n(n-1)\frac{V}{8\pi G a^2}, \quad T_H = -\frac{\dot{H}}{2\pi H}. \tag{6.7}$$

and we have separated the energy into a matter part and a cosmological constant part, i.e., $E = E_m + E_{\text{cosm}}$, where $E_{\text{cosm}} = \frac{\Lambda}{8\pi G}V$. This is simply a way to rewrite the FRW equations in a form that resembles the equation defining the entropy of a two-dimensional CFT. However, the following remark is in order: the presence of cosmological constant may

[6] In our discussion of cosmology based on Einstein gravity, we parametrize the curvature of the spatial hypersurfaces in terms of $k = -1, 0, 1$ since direct comparison with the standard cosmological equations is then straightforward.

change some of the coefficients in eq. (6.6) and this depends on precisely how the separation between the strongly and weakly interacting gravitational phases is made (compare with [30, 31]). In any case, the energy associated with the cosmological constant term is hidden in the expression for E, eq. (6.6).

Eq. (6.5) may also be rewritten in another form:

$$S_H^2 = S_{BH}\left(2S_B - kS_{BH}\right) . \tag{6.8}$$

Since the definition of S_B normally contains only matter contributions, it is reasonable to define $S_B \equiv S_B^m + S_B^{\text{cosm}}$, where the entropy associated with the cosmological constant is given by

$$S_B^{\text{cosm}} = \frac{aV\Lambda}{4nG} . \tag{6.9}$$

The appearance of such a new "cosmological constant" contribution to the entropy in the CV formula is quite remarkable.

Thus far, we have discussed the appearance of the CV formula as a way to rewrite the FRW equations. However, the CV formula appears in a second formulation when one calculates the entropy, S, of the universe. Indeed, following ref [24], one can represent the total energy $E = \rho V$ of the universe as the sum of the extensive energy, E_E, and the subextensive (Casimir) energy E_C:

$$E(S,V) = E_E(S,V) + \frac{1}{2}E_C(S,V) . \tag{6.10}$$

Note that unlike the case considered by Verlinde [24], the cosmological constant contribution appears in E_E. Nevertheless, the constant rescaling of the energy is given by

$$\begin{aligned} E_E\left(\lambda S, \lambda V\right) &= \lambda E_E\left(S, V\right) , \\ E_C\left(\lambda S, \lambda V\right) &= \lambda^{1-\frac{2}{n}} E_C\left(S, V\right) . \end{aligned} \tag{6.11}$$

Now, if we assume that the first law of thermodynamics is valid and that the expansion is adiabatic, we deduce that

$$dS = 0 , \quad s = \frac{a^n}{T}\left(\rho + p\right) + s_0 , \tag{6.12}$$

where the entropy $S \equiv s \int d^n x \sqrt{g}$, s_0 is an integration constant and T is the temperature of the universe. It then follows that the Casimir energy is given by [29]

$$E_C = n\left(E + pV - TS\right) = -nTs_0 \int d^n x \sqrt{g} \tag{6.13}$$

and, consequently, that $E_C \sim a^{-n\omega}$ and $E_E - E_{\text{cosm}} \sim a^{-n\omega}$. This further implies that the products $E_C a^{n\omega}$ and $\left(E_E - E_{\text{cosm}}\right) a^{n\omega}$ are independent of the spatial volume of the universe, V. By employing the scaling relations (6.11) one then concludes that [29]:

$$E_E - E_{\text{cosm}} = \frac{\alpha}{4\pi a^{n\omega}} S^{\omega+1} , \qquad E_C = \frac{\beta}{2\pi a^{n\omega}} S^{\omega+1-\frac{2}{n}} , \tag{6.14}$$

where α and β are arbitrary constants. Hence, the entropy is given by

$$S = \left[\frac{2\pi a^{n\omega}}{\sqrt{\alpha\beta}} \sqrt{E_C \left(E_E - E_{\rm cosm} \right)} \right]^{\frac{n}{(\omega+1)n-1}} . \tag{6.15}$$

Eq. (6.15) represents the generalization of the Cardy-Verlinde formula found by Youm [29] in the absence of a contribution from the cosmological constant. The negative term associated with such a cosmological entropy is quite remarkable. In the case of a radiation-dominated universe, eq. (6.15) reduces to the standard CV formula with the familiar square root term.

We may now proceed to consider whether a similar formulation of the Friedmann equation (3.8) is possible for the Einstein-GB-Maxwell braneworld model. We may rewrite eq. (3.8) in the form:

$$\begin{aligned} H^2 &= -\frac{k}{2a^2} + \frac{\kappa_4^2}{6} \frac{\tilde{E}}{V} , \\ \tilde{E} &= \frac{6V_3 a^3}{\kappa_4^2} \left[\frac{\mathcal{G}^2}{\mathcal{H}^2} - \frac{1}{2a^2} \left\{ \frac{a^2}{2\epsilon l^2} \pm \frac{Y^{1/2}}{c} \right\} \right] , \\ V &= a^3 V_3 , \end{aligned} \tag{6.16}$$

where V_3 is the volume of the three-dimensional sphere with unit radius and κ_4 is the four-dimensional gravitational coupling.

In [24], it was shown that the standard FRW equation in d dimensions can be regarded as a d-dimensional analogue of the Cardy formula for a two-dimensional CFT [25]:

$$\tilde{\mathcal{S}} = 2\pi \sqrt{\frac{c}{6} \left(L_0 - \frac{k}{d-2} \frac{c}{24} \right)} , \tag{6.17}$$

where c is the analogue of the two-dimensional central charge and L_0 is the analogue of the two-dimensional hamiltonian. In the present case, making the identifications

$$\begin{aligned} \frac{2\pi \tilde{E} r}{d-1} &\to 2\pi L_0 , \\ \frac{(d-2)V}{\kappa_d^2 r} &\to \frac{c}{24} , \\ \frac{4\pi(d-2)HV}{\kappa_d^2} &\to \tilde{\mathcal{S}} , \end{aligned} \tag{6.18}$$

implies that the FRW-like equation (6.16) has the same form as eq. (6.17). This is the first analogy that has been made with the CV formula, when one rewrites the FRW equation in a form similar to that of the Cardy formula.

It is also interesting to develop a relationship with thermodynamics. In [23], it was shown that the thermodynamical energy, E, and entropy, $\mathcal{S}$, of the bulk spacetime are given by

$$E = \frac{3l^2}{16\kappa^2} V_3 \left(1 - 12\epsilon\right) \left(k^2 + \frac{16\kappa^4 \tilde{\mu}}{l^4} \right) , \tag{6.19}$$

$$\mathcal{S} = \frac{V_3}{\kappa^2} \left(\frac{1 - 12\epsilon}{1 - 4\epsilon} \right) \left(4\pi r_H^3 + 24\epsilon k \pi r_H \right) + \mathcal{S}_0 , \tag{6.20}$$

where r_H is the black hole radius, defined by the condition that $\mathrm{e}^{2\nu}$ in eq. (2.5) vanishes at $r = r_H$. This constraint reduces to

$$0 = r_H^6 - \frac{kl^2 r_H^4}{2\,(2\epsilon - 1)} - \frac{\kappa^4 \tilde{\mu}\,(4\epsilon - 1)\, r_H^2}{2\epsilon - 1} - \frac{\tilde{Q}^2 \kappa^4}{2\,(2\epsilon - 1)} \tag{6.21}$$

and solving eq. (6.21) with respect to $\tilde{\mu}$, we find that

$$\tilde{\mu} = \frac{1}{2\kappa^4}\left(k\epsilon - \frac{1}{2}\right)^{-1}\left\{(2\epsilon - 1) r_H^4 - k r_H^2 l^2 - \frac{\tilde{Q}^2 \kappa^2 l^2}{2c r_H^2}\right\}. \tag{6.22}$$

It is unclear from the above relations how to relate the Friedmann equation to the thermodynamics of the bulk black hole through a CV formula. In order to understand this issue more fully, therefore, we now consider a simpler five-dimensional theory that has a dual quantum field theory (QFT) analogue. Specifically, instead of the Gauss-Bonnet theory, we can begin with the following five-dimensional, higher-order gravity theory:

$$S = \int d^{d+1}x \sqrt{-\hat{G}}\left\{\hat{a}\hat{R}^2 + \hat{b}\hat{R}_{\mu\nu}\hat{R}^{\mu\nu} + \frac{1}{\kappa^2}\hat{R} - \Lambda\right\}, \tag{6.23}$$

where $\{\hat{a}, \hat{b}\}$ are coupling constants. In this case, the Schwarzschild-anti de Sitter (SAdS) space is an exact solution [12]:

$$\begin{aligned} ds^2 &= \hat{G}_{\mu\nu} dx^\mu dx^\nu \\ &= -\mathrm{e}^{2\rho_0} dt^2 + \mathrm{e}^{-2\rho_0} dr^2 + r^2 \sum_{i,j}^{d-1} g_{ij} dx^i dx^j\,, \\ \mathrm{e}^{2\rho_0} &= \frac{1}{r^{d-2}}\left(-\mu + \frac{k r^{d-2}}{d-2} + \frac{r^d}{l^2}\right), \end{aligned} \tag{6.24}$$

where μ parametrizes the mass of the black hole and the scale parameter, l, is determined by solving the constraint equation:

$$0 = \frac{80\hat{a}}{l^4} + \frac{16\hat{b}}{l^4} - \frac{12}{\kappa^2 l^2} - \Lambda \tag{6.25}$$

and the horizon radius, r_h, is deduced by solving the equation $e^{2\rho_0(r_H)} = 0$ in (6.24), i.e.,

$$r_H^2 = -\frac{kl^2}{4} + \frac{1}{2}\sqrt{\frac{k^2}{4} l^4 + 4\mu l^2}\,. \tag{6.26}$$

The Hawking temperature, T_H, is then given by

$$T_H = \frac{(e^{2\rho})'|_{r=r_H}}{4\pi} = \frac{k}{4\pi r_H} + \frac{r_H}{\pi l^2}\,, \tag{6.27}$$

where a prime denotes differentiation with respect to r. One can also rewrite the mass parameter, μ, using r_H or T_H from eq. (6.26) as follows:

$$\begin{aligned} \mu &= \frac{r_H^4}{l^2} + \frac{k r_H^2}{2} = r_H^2\left(\frac{r_H^2}{l^2} + \frac{k}{2}\right) \\ &= \frac{1}{4}\left(\pi l^2 T_H \pm \sqrt{(\pi l^2 T_H)^2 - kl^2}\right)^2 \left(\frac{1}{4l^2}\left(\pi l^2 T_H \pm \sqrt{(\pi l^2 T_H)^2 - kl^2}\right)^2 + \frac{k}{2}\right). \end{aligned} \tag{6.28}$$

The entropy $\mathcal{S}$ and the thermodynamical energy E of the black hole are given by [12]

$$\begin{aligned}\mathcal{S} &= \frac{V_3 \pi r_H^3}{2}\left(\frac{8}{\kappa^2} - \frac{320\hat{a}}{l^2} - \frac{64\hat{b}}{l^2}\right) \\ &= \frac{V_3\pi}{16}\left(\pi l^2 T_H \pm \sqrt{(\pi l^2 T_H)^2 - kl^2}\right)^3 \left(\frac{8}{\kappa^2} - \frac{320\hat{a}}{l^2} - \frac{64\hat{b}}{l^2}\right). \end{aligned} \tag{6.29}$$

$$E = \frac{3V_3\mu}{8}\left(\frac{8}{\kappa^2} - \frac{320\hat{a}}{l^2} - \frac{64\hat{b}}{l^2}\right). \tag{6.30}$$

On the other hand, the FRW equations of the brane universe in the SAdS background are given by

$$\begin{aligned} H^2 &= -\frac{k}{2a^2} + \frac{\kappa_4^2}{6}\frac{\tilde{E}}{V}, \qquad \dot{H} = -\frac{\kappa_4^2}{4}\left(\frac{\tilde{E}}{V} + p\right) + \frac{k}{2a^2}, \\ \tilde{E} &= \frac{6\mu V_3}{\kappa_4^2 a}, \qquad p = \frac{2\mu}{a^4\kappa_4^2}, \qquad V = a^3 V_3, \\ \frac{1}{\kappa_4^2} &= \frac{l}{2}\left(\frac{1}{\kappa^2} - \frac{40\hat{a}}{l^2} - \frac{8\hat{b}}{l^2}\right). \end{aligned} \tag{6.31}$$

Since $3p = \tilde{E}/V$, the trace of the energy-stress tensor arising from the matter on the brane vanishes, i.e., $T^{\text{matter}\ \mu}{}_{\mu} = 0$. Thus, the matter on the brane can be regarded as radiation or, equivalently, as massless fields. In other words, the field theory on the brane should be a conformal theory.

We now assume that the total entropy $\mathcal{S}$ of the CFT on the brane is given by eq. (6.29). If this entropy is constant during the cosmological evolution, the entropy density s is given by

$$s = \frac{\mathcal{S}}{a^3 V_3} = \frac{4\pi r_H^3}{\tilde{\kappa}^2 a^3} = \frac{8\pi r_H^3}{l\kappa_4 a^3}. \tag{6.32}$$

If we further assume that the temperature T on the brane differs from the Hawking temperature T_H by the factor l/a (for a discussion of why such a rescaling is necessary, see [32]), it follows that

$$T = \frac{l}{a}T_H = \frac{r_H}{\pi a l} + \frac{kl}{4\pi a r_H} \tag{6.33}$$

and, when $a = r_H$, this implies that

$$T = \frac{1}{\pi l} + \frac{k}{4\pi r_H^2}. \tag{6.34}$$

If the energy and entropy are purely extensive, the quantity $\tilde{E} + pV - T\mathcal{S}$ vanishes. In general, this condition does not hold and one can define the Casimir energy E_C as in the above case for Einstein gravity:

$$E_C = 3\left(\tilde{E} + pV - T\mathcal{S}\right). \tag{6.35}$$

Then, by using eqs. (6.31), (6.32), and (6.33), we find that

$$E_C = \frac{6kr_H^2 V}{\kappa_4^2 a^4} = \frac{6kr_H^2 V_3}{\kappa_4^2 a} = \frac{3lkr_H^2 V_3}{\kappa_4^2 a}\left(1 - \frac{40\hat{a}\kappa^2}{l^2} - \frac{8\hat{b}\kappa^2}{l^2}\right). \tag{6.36}$$

Consequently, the Casimir energy vanishes for $k = 0$. When $\hat{a}$ and $\hat{b}$ are small, E_C is positive (negative) for $k = 2$ ($k = -2$). If, on the other hand, either $\hat{a}$ or $\hat{b}$ is large and positive, E_C can be negative (positive) even when $k = 2$ ($k = -2$). Finally, by combining eqs. (6.29), (6.31), and (6.36), for the case where $k \neq 0$, we find that

$$\mathcal{S} = \frac{4\pi a}{3\sqrt{|2k|}}\sqrt{\left|E_C\left(\tilde{E} - E_C\right)\right|}. \tag{6.37}$$

Eq. (6.37) has precisely the same form as the corresponding equation that arises in Einstein gravity. Thus, we have demonstrated how the FRW equation, when written in the CV form, can be related to the thermodynamics of the bulk black hole. However, in the theory where the GB term is present, it is anticipated that the dual QFT (if it exists) will not be a conformal theory. In this case, an extension of AdS/CFT correspondence to include a non-CFT will be required and it is not clear how the FRW equation can be related to black hole thermodynamics in this scenario. It is possible that strong differences between the two approaches may arise.

7. Discussion

In this paper we have derived the brane Friedmann equation for the general, vacuum FRW brane embedded in five-dimensional Einstein-GB-Maxwell gravity, where the bulk spacetime may be interpreted as a charged black hole in an asymptotically AdS (or dS) space.

Recently, brane dynamics of the second Randall-Sundrum model [5] has been discussed within the framework of Einstein-GB gravity [15, 16, 17]. Our approach is different to these previous works because we have considered the surface terms that must be introduced into the bulk action (2.1) to ensure that the variational principle is well-defined. Moreover, our boundary action is fixed by the finiteness of the bulk space, or equivalently, by cancelling the leading-order divergences in the asymptotically AdS bulk.

It is well known that in Einstein gravity, the variational principle is ill-defined when the manifold has a boundary, since the scalar curvature contains second-order derivatives of the metric, $g_{\mu\nu}$. Consequently, when the action is varied with respect to the metric, it acquires a non-trivial term that is proportional to the derivative of $\delta g_{\mu\nu}$ with respect to the coordinate that is transverse to the boundary. This term is cancelled by introducing the Gibbons-Hawking surface term [27] into the action. In this paper, the analogous counterterms for the GB contribution were introduced, together with the Gibbons-Hawking term and the leading counterterm arising from the brane vacuum energy. The crucial point is that once the action is varied, terms in $\delta A_{,z}$ no longer arise, thus ensuring that the variational principle is now well-defined [12]. Extremizing the action then led to the functional form of $A_{,z}$ and, hence, the effective Friedmann equation (3.8) on the brane. We emphasize that the introduction of the boundary terms is not *ad hoc* and that their functional form is determined directly from the requirement that the variational principle be well defined and that the bulk space should be finite.

When the higher-order Gauss-Bonnet terms are present, the derivative terms in eq. (3.2) are non-trivial because the bulk spacetime is no longer an Einstein space. This leads to the extra complications in the form of $A_{,z}$, as summarized in eq. (3.6), once the expressions (3.3) and (3.4) for the Ricci scalar and tensor are substituted.

Consequently, the form of the Friedmann equation (3.8) is itself highly non-trivial. Nevertheless, the qualitative dynamics can be established. The condition (4.2) may be imposed on the parameters of the model by requiring the bulk action to be finite. When this condition is satisfied, the effective cosmological constant on the brane vanishes for the negative-branch cosmologies. The main conclusion is that although the charge and mass of the bulk black hole are negligible at large spatial volumes, they radically alter the dynamics as the scale factor diminishes in size. In particular, the Friedmann equation is dominated by a negative term in the limit of vanishing spatial volume, implying that such a limit is unphysical.

Finally, we discussed two approaches where a (generalized) Cardy-Verlinde formula arises in FRW cosmology. The first approach rewrites the FRW equations in a suitable form and this method may be adapted to the GB braneworld model considered in this paper. In the second approach, the entropy of the universe is calculated in terms of a dual quantum field theory. However, at present, it is not clear how this technique may be employed within the context of the GB braneworld scenario, since it is expected that the QFT that is dual to the AdS black hole should be non-conformal in the presence of a GB term.

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A. Components of the Riemann tensor

When the five-dimensional metric has the form given in eqs. (2.10) and (2.11), the non-vanishing components of the curvature tensor and its contractions are:

$$\begin{aligned}
R_{zizj} &= e^{2A}\left(-A_{,zz} - (A_{,z})^2\right)\tilde{g}_{ij} \\
R_{zA\sigma B} &= -l^2 e^{2A} A_{,z\sigma} \overset{s}{g}_{AB} \\
R_{\sigma A\sigma B} &= \left(-l^2 e^{2A} A_{,\sigma\sigma} - l^2 e^{4A}(A_{,z})^2\right)\overset{s}{g}_{AB} \\
R_{ABCD} &= \left(l^2 e^{2A} - l^2 e^{2A}(A_{,\sigma})^2 - l^4 e^{4A}(A_{,z})^2\right)\left(\overset{s}{g}_{AC}\overset{s}{g}_{BD} - \overset{s}{g}_{AD}\overset{s}{g}_{BC}\right) \\
R_{zz} &= 4\left(-A_{,zz} - (A_{,z})^2\right)
\end{aligned}$$

$$
\begin{aligned}
R_{z\sigma} &= -3A_{,z\sigma} \\
R_{\sigma\sigma} &= l^2 \mathrm{e}^{2A} \left(-A_{,zz} - 4\left(A_{,z}\right)^2 \right) - 3A_{,\sigma\sigma} \\
R_{AB} &= \left(l^2 \mathrm{e}^{2A} \left(-A_{,zz} - 4\left(A_{,z}\right)^2 \right) - A_{,\sigma\sigma} - 2\left(A_{,\sigma}\right)^2 + 2 \right) g^s_{AB} \\
R &= -8A_{,zz} - 20\left(A_{,z}\right)^2 + l^{-2}\mathrm{e}^{-2A} \left(-6A_{,\sigma\sigma} - 6\left(A_{,\sigma}\right)^2 + 6 \right),
\end{aligned}
\tag{A.1}
$$

where g^s_{AB} represents the metric on the three-space with the line-element given by $d\Omega^2_{k,3}$ in eq. (2.11).

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